Evidence from a Pandemic: The Importance of Developing Students' Schemes for Making Comparisons of Relative Size

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Introduction

By March 2020, citizens around the world were being inundated with data about the COVID-19 pandemic in the form of graphs, predictive models, probabilities, and multiple statistics relating to deaths, hospitalizations, and risk of infection. One hypothesis we generated based on our research team's background in mathematics education research was that citizens' meanings for relative size concepts (such as fractions, percentages, and rate of change) were important for their understanding of COVID-19 data (Byerley & Thompson, 2017; Byerley, 2019). To investigate our hypothesis, we interviewed 32 citizens on how they interpreted COVID-19 data and data representations we gathered from the media. This report shares our observations about citizens' comparisons of quantities related to COVID-19 on one item. Our findings support arguments made in prior mathematics education research about the importance of people developing the ability to compare the relative size of two quantities.

Background Literature

Thompson and Saldanha (2003) argued that "how students understand a concept has important implications for what they can do and learn subsequently" (p. 95). Their 2003 paper gives example of different ways students might understand fractions, and the constraints and affordances of different understandings of fractions. Their paper summarizes the vast amount of evidence that developing understanding of fractions (and related concepts like rate and percent) as tools to compare the relative sizes of quantities is a difficult goal that many US citizens have not achieved. They also referenced other mathematics education research that helped them make distinctions among various meanings for fractions including Steffe and Olive (2009).

Thompson and Saldanha (2003) contrasted two meanings for fractions, *part-whole* and *relative size.* A common way of expressing a *part-whole* meaning for a fraction such as 4/5 is to say "4 out of 5." This meaning is often helpful when the numerator is smaller than the denominator and when the quantity the numerator represents is a subset of the denominator. For example, it makes sense to use a part-whole meaning to say, "Today my husband told me that 13 out of his 200 students have tested positive for COVID-19." The students in his classes who have tested positive is necessarily a subset of the entire group of students.

There are other situations where thinking of fractions as a comparison of the *relative size* of two quantities is more productive than the part-whole meaning. Thompson and Saldanha wrote about the productiveness of students developing relative size meanings for fractions that were strongly connected to quantitative meanings for proportion, multiplication, and division. Consider someone who wants to compare infection fatality rates (IFR) from COVID-19 (let's assume 0.6), to infection fatality rates of the flu (let's assume 0.1) (see Figure 1). A person with a part-whole meaning would have a hard time making sense of "0.6 out of 0.1". This is because a COVID-19 IFR is not a subset of a flu IFR and students with a primarily part-whole meaning for fractions often think the numerator (0.6) should be smaller than the denominator (0.1). There are other productive ways to think about the number 0.6/0.1. For example, we can compare 0.6 and 0.1 multiplicatively and say that since it takes six copies of one-tenth to make six-tenths, that 0.6 is *6 times as much as* 0.1. Thus the IFR from COVID-19 is six times as large as the IFR from the flu. Steffe and Olive's (2009) research shows that it is cognitively easier to develop part-whole meanings for fractions and that students typically develop relative size meanings later after they are able to mentally coordinate three different units at one time. To develop a part-whole meaning the student only needs to coordinate the size of the part and the whole simultaneously.

Methods

We conducted task-based clinical interviews with 25 United States (US) citizens and 7 South Korea (SK) citizens between April 2nd, 2020 and May 11th, 2020 (Ginsburg, 1997; Goldin, 1997). We included people from two countries because the governments' response to COVID-19 varied widely by location (Solano, Maki, Adirim, Shih, & Hennekens, 2020). Here we present citizen's responses to one item (Figure 1).

> Scientists (such as Wu and team) estimate the death rate for COVID-19 is between 0.66% and 2.1%. The death rate for the seasonal flu is usually about 0.1% in the U.S.
>
> a. How should this data impact decision making about social distancing?
>
> b. Suppose there are two hypothetical situations.
> In one situation 50 million people get the flu. In the other situation 50 million people get the coronavirus. Assuming the death rates of 0.1% and 2.1% how many times as many people will die from the coronavirus as the flu.

Figure 1. Interview item named "Flu versus COVID-19 rates".

Based on prior research we hypothesized that some citizens would use a part-whole meaning for fractions to first find 2.1% of 50 million and 0.1% of 50 million and then compare 105,000 deaths to 5,000 deaths. We also suspected that some citizens would immediately compare the relative size of 2.1 and 0.1 and realize that they could ignore the information about 50 million. Finally, we wondered if some people would think 2.1% was a small percent because 2.1% is considered small in many contexts such as giving tips at restaurants and credit card interest rates.

Summary of Interviews

Thirty one out of thirty two people interviewed responded to "Flu vs. COVID-19 rates" and their responses are summarized in Table 1. Note that some people responded in more than one way so the subtotals do not add to 31 people. Overall, we found that people assessed the relative severity of COVID-19 and the flu using many types of information and that citizens' correct comparison of percentages in the problem was neither necessary or sufficient to decide that COVID-19 is more severe than a typical seasonal flu.

Table 1. Summary of responses to "Flu vs. COVID rates."

“Flu vs. COVID-19 rates”						
	Approximately correct multiplicative comparison.	Incorrect multiplicative comparison.	Asked to make multiplicative comparison but citizen didn’t respond	Said 2% of a large number is very large.	Said 2.1% and 0.1% are both small so COVID-19 is not too serious	Said scientists incorrectly estimated infection fatality rates for COVID-19.
Flu is more severe than COVID-19	1	1	2	3	1	2
COVID-19 is more severe than flu	12 incl. Eunseok and Amelia	9	1	12 incl. Eunseok and Amelia	1	0
Unsure if flu or COVID-19 is worse	0	0	1	2	0	0
Subtotal	13	10	4	17	2	2

Examples of Part-Whole and Relative Size Meanings

We provide examples from Eunseok and Amelia that show how people with different methods of comparing quantities approached the problem.

Eunseok (SK) felt COVID-19 was more severe than the flu. He used a part-whole meaning for percentage to reason about the infection fatality rates. When ask how he compared 2.1 and 0.1 percent he said:

> *2.1% indicates, if there are 100 people, for example, two people die, and one person dies if there are 1,000 people who were infected with flu. In other words, the coronavirus kills 21 people when 1,000 people are infected, and the flu kills one person if 1,000 people were infected.*

Eunseok not only understood 2% as 2 out of 100, but also was able to reason proportionally to determine that 2.1% means 21 people out of 1000. The conversation continued with Eunseok reasoning:

> *0.1% of 50 million is 1/1000 of 50 million. Then… isn't it 50,000?*
> *1 million people would die when death rate of the coronavirus is 2.1% since it is like 2/100. Then isn't that 20 times more?*

Eunseok used his previous work of “21 people out of a thousand” and “1 person out of a thousand” to compare 2.1% and 0.1%. He did not immediately respond that 2.1% is 21 times as large as 0.1%, but instead took intermediate estimation steps. Though his estimate of 20 was slightly inaccurate, Eunseok did have the resources necessary to compare infection fatality rates.

Amelia also was able to compare infection fatality rates but did so using different reasoning than Eunseok. She responded:

> *The numbers of 50 million and 50 million do not matter. You’re comparing 0.1 to 2.1. You could be saying you are comparing one to 21. I guess we’ll take 21 times. We’ll take 21 times as many.*

Notice that Amelia's response was extremely efficient. Unlike many other US citizens she did not need a calculator, the internet, or a piece of paper to solve the problem suggesting she had developed the ability to coordinate multiple relationships almost simultaneously by directly comparing the relative size of the percentages themselves. Amelia had an understanding of place value that allowed her to see that 10 copies of 0.1 was equal to one, and 2.1 was 21 copies of 0.1. She understood that comparing 2.1(x) to 0.1(x), where x is any number, is the same as comparing the relative size of 2.1 and 0.1. Amelia was able to rapidly coordinate 0.1%, 2.1% and the whole, 50 million, to recognize that she did not need to consider the 50 million.

We see ways of reasoning that enable citizens to almost effortlessly compare relative sizes as productive for citizens' understanding of media data, because a typical news segment or article has so much information that the citizen is quickly carried along to the next piece of information. Many of the citizens we interviewed were much less inclined than Amelia to make a comparison because the process of comparing two quantities was cumbersome for them.

We agree with Thompson and Saldanha (2003) that helping students develop interconnected and powerful quantitative meanings for multiplication, division, proportions, percentages and fractions is critical for positioning them to efficiently make sense of the world. We view helping student develop reasoning like Amelia's as a great goal for teachers. On the other hand, we acknowledge that Eunseok was able to reach the same correct conclusion and each step in Eunseok's process would likely be understandable for citizens who think of fractions as parts out of whole, but not as comparisons of relative size. Eunseok's reasoning using part-whole meanings was more common in our sample and many citizens could likely make sense of COVID-19 data more easily if each step of Eunseok's process was described. Currently many media and governmental sources only provide percentages without supporting citizens in making sense of what the percentages convey, and we encourage media and government sources to scaffold their information more in order to help citizens with primarily part-whole meanings fully understand their data.

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